ISBN : 978-93-91077-53-2

Pre-Conference Proceedings of the International Conference on Discrete Mathematics (ICDM2021-MSU)

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ISBN 978-93-91077-53-2
9 789391 077532

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The 3-Component Connectivity Number of Arithmetic Graph

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Abstract

A 3-component cut of G is a set of vertices whose removal yields a graph with at least three connected components. The 3-component connectivity number of G is denoted as $\kappa_3(G)$ is the cardinality of minimum number of vertices that must be removed from G in order to obtain a graph with at least three connected components. In this paper, we identified that for an arithmetic graph $G = V_n$ where $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$, $r \geq 3$ and if $a_i > 2$ for at least one i then the 3-component connectivity number is equal to its connectivity number r.

Keywords: arithmetic graph, component connectivity number, component cut.
Subject Classification: 05C40

1 Introduction

For notation and graph theory terminology not given here, we follow [2]. In this paper three component connectivity of an Arithmetic Graph $G = V_n$ is studied. A 3-component cut of G is a set of vertices whose removal yields a graph with at least three connected components. The three component connectivity number of G is denoted as $\kappa_3(G)$ is the cardinality of minimum number of vertices that must be removed from G in order to obtain a graph with at least three connected components. This concept was originally introduced by sampathkumar [10] has been recently studied for hypercubes by Hsu-et-al-in [11]. The definition is from [3]. A 3-component cut of G is a set of vertices whose removal yields a graph with at least three connected components. The 3- component connectivity number of G is denoted as $\kappa_3(G)$ is the cardinality of minimum number of vertices that must be removed from G in order to obtain a graph with at least three connected components. The arithmetic graph V_n is defined as a graph with its vertex set is the set consists of the divisors of n(excluding1) where n is a positive integer and $G = V_n, n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$ where $p_i's$ are distinct primes and $a_i's \geq 1$ and two distinct vertices a,b which are not of the same parity are adjacent in this graph if $(a,b) = p_i$ for some i, $1 \leq i \leq r$. The vertices a and b are said to be of the same parity if both a and b are the powers of the same prime, for instance $a = p^2$, $b = p^5$. This concept was studied from [12]. Also various authors studied different parameters of an arithmetic graph. In [7] the super connectivity number of an arithmetic graph is studied by L.Mary jenitha and S.Sujitha. In [5] the connectivity number of an arithmetic graph is studied by L.Mary jenitha and S.Sujitha. Later, the various parameters of connectivity of an arithmetic graph are studied by the same authors in [6, 8]. The following theorems are used in sequel.

Theorem 1.1. [6]*For an arithmetic graph* G=V_n, n =$p_1^{a_1} \times p_2^{a_2}$ *where* p_1 *and* p_2 *are distinct primes,* $a_1, a_2 \geq 1$ *then* $\epsilon = 4a_1a_2 - a_1 - a_2$, *where* ϵ *is the size of the graph* G.

Theorem 1.2. [6]*For an arithmetic graph* G=V_n, n =$p_1^{a_1} \times p_2^{a_2}$ *where* p_1 *and* p_2 *are distinct primes,* $a_1, a_2 \geq 1$ *then* G *is a bipartite graph.*

Theorem 1.3. [9] *For an arithmetic graph $G = V_n$, $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$, then the number of vertices of G is $|V| = \prod_{i=1}^{r}(a_i + 1) - 1$.*

Theorem 1.4. [6] *Let $G = V_n$ an arithmetic graph $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$, for any vertex $u = \prod_{i\in B} p_i^{\alpha_i}$ where $B \subseteq 1, 2, 3, \ldots r, 1 \le \alpha_i \le a_i \forall i \in B$.*

(1) If $u = p_j$ where $j \in 1, 2, 3, \ldots, r$, then $deg(u) = \left[a_j \prod_{i=1,i\neq j}^{r} (a_i + 1) - 1\right] - |a_j - 1|$.

(2) If $u = p_i^{\alpha_i}$ $1 < \alpha_i \le a_i \forall i \in B$, then $deg(u) = [\prod_{i=1,i\notin B}^{r} (a_i + 1)] - 1$

(3) If $u = \prod_{i\in B} p_i^{\alpha_i}, |B| \ge 2, 1 < \alpha_i \le a_i, \forall i \in B$ then $deg(u) = |B| \prod_{i=1,i\notin B}^{r} (a_i + 1)$

(4) If $u = \prod_{i\in B} p_i^{\alpha_i}, \alpha_i = 1$ for some $i \in B' \subseteq B$, then $deg(u) = [\left|B - B'\right| + \sum_{i\in B'} a_i] \prod_{i=1,i\notin B}^{r} (a_i + 1)$ where B is the number of primes product in u, B' is the number of primes having power 1 in chosen vertex u.

Observation (1.5). [10] The following are the inequalities of an arbitrary simple non complete graph G of order n.
(i) $\kappa_r(G) \le \kappa_{r+1}(G)$, for $r = \{2, 3, \ldots, n - 1\}$.
(ii) $\lambda_r(G) \le \lambda_{r+1}(G)$, for $r = \{2, 3, \ldots, n - 1\}$.

Theorem 1.6. [5] *For an arithmetic graph $G= V_n$, $n =p_1^{a_1} \times p_2^{a_2}$ where p_1 and p_2 are distinct primes, then*

$$\kappa(V_n) = \kappa'(V_n) = \begin{cases} 1 & for\ a_i = 1\ a_j > 1; i, j = 1, 2 \\ 2 & for\ a_i > 1; i = 1, 2 \end{cases}$$

Theorem 1.7. [5] *For an arithmetic graph $G = V_n, n = p_1^{a_1} \times p_2^{a_2} \times \ldots \times p_r^{a_r}$ where $p_i, i = 1, \ldots, r(r > 2)$ are distinct primes and $a_i = 1$ for all $i = 1, 2, \ldots, r$ then $\kappa(V_n) = \kappa'(V_n) = r$.*

Theorem 1.8. [5] *For an arithmetic graph $G = V_n, n =p_1^{a_1} \times p_2^{a_2} \times \ldots\ldots \times p_r^{a_r}$ where $p_1, p_2, \ldots, p_r$ are distinct primes and $a_i's \ge 1$ for all $i = 1, 2, 3, \ldots, r$ and n is a product of more than two primes, then $\kappa(V_n) = \kappa'(V_n) = r$.*

2 The 3-Component Connectivity Number of $G = V_n$

In this section the 3- component connectivity number of an arithmetic graph $G = V_n$, where $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$, $a_i \ge 1$, $r \ge 2$ for $i \in \{1, 2, \ldots, r\}$ are categorised.

Theorem 2.1. *A 3-component cut does not exist for an arithmetic graph $G = V_n$, where $n = p_1^{a_1} \times p_2^{a_2}$; $a_1 = a_2 = 1$.*

Proof. Consider the arithmetic graph $V_n, n = p_1^{a_1} \times p_2^{a_2}$ $a_1 = a_2 = 1$, then n is the product of two distinct primes. By Theorem1.3, we have $|V(G)| = 3$. Obviously it is clear that there does not exist a three component cut. □

Theorem 2.2. *If $G = V_n$ is an arithmetic graph where $n = p_1^2 \times p_2$ where p_1 and p_2 are distinct primes then $\kappa_3(G) = 2$.*

Proof. Let $G = V_n$ be an arithmetic graph where $n = p_1^2 \times p_2$ then the vertex set consists of vertices $p_1, p_1^2, p_2, p_1 \times p_2, p_1^2 \times p_2$. Here the only pendant vertex in G is p_1^2 and $N(p_1^2) = p_1 \times p_2$. So the removal of the vertex $p_1 \times p_2$ makes the graph disconnected into two components G_1 and G_2 where G_1 is an isolated vertex p_1^2 and G_2 is a path with three vertices $p_1, p_2, p_1^2 \times p_2$. Since $(p_1, p_1^2 \times p_2) = p_1$ and $(p_2, p_1^2 \times p_2) = p_2$, the vertex $p_1^2 \times p_2$ is the internal vertex of the path. So, the removal of the vertex $p_1^2 \times p_2$ from G_2 results the component disconnected into two isolated vertices p_1 and p_2. Since the induced graph of $G - S$ has three isolated vertices, the set $S = \{p_1 \times p_2, p_1^2 \times p_2\}$ is a 3-component cut. Also, it is clear that no set $S_1 \subset S$ is a 3-component cut, hence S is minimum and we have $\kappa_3(G) = |S| = 2$. □

Theorem 2.3. *In an arithmetic graph $G = V_n$ $\kappa_3(G) = 1$ iff $n = p_1^{a_1} \times p_2$ where $p_1,\ p_2$ are distinct primes and $a_1 > 2$.*

Proof. Let $G = V_n$ be an arithmetic graph. Suppose that, the 3-connectivity number, $\kappa_3(G) = 1$. From the Observation 1.5, $\kappa(G) \leq \kappa_3(G)$ and hence $\kappa(G) = 1$. By Theorem 1.6, the connectivity number $\kappa(G) = 1$ for $n = p_1^{a_1} \times p_2$ but if $a_1 = 1$, then G is a tree with three vertices and if $a_1 = 2$ then by Theorem 2.1 $\kappa_3(G) = 2$. Therefore, the only possibility is $n = p_1^{a_1} \times p_2$ where $a_1 > 2$. Conversely, consider an arithmetic graph $G = V_n$ $n = p_1^{a_1} \times p_2$ where $a_1 > 2$. To prove that $\kappa_3(G) = 1$ (i.e) to prove the removal of exactly one vertex makes the graph disconnected into three or more components. By Theorem 1.6, the connectivity number $\kappa(G) = 1$ for $n = p_1^{a_1} \times p_2$; $a_1 \geq 1$. By the proof of the Theorem 1.1 and by the definition of an arithmetic graph, $(p_1^{\alpha_1}, p_1 \times p_2) = p_1$; $1 \leq \alpha_1 \leq a_1$ and $N(p_1^{\alpha_1}) = \{p_1 \times p_2\}$ for $1 < \alpha_1 \leq a_1$ thus it is clear that the number of pendant vertices in G is $a_1 - 1$, and the neighbor for these pendant vertices is a unique vertex $p_1 \times p_2$. Hence $S = \{p_1 \times p_2\}$ is a 3-component cut. □

Theorem 2.4. *For an arithmetic graph $G = V_n, n = p_1^{a_1} \times p_2^{a_2}$ where $a_1 = a_2 = 2$ then $\kappa_3(G) = 3$.*

Proof. By Theorem 1.6 $\kappa(G) = 2$ and $S_1 = \{p_1, p_2\}$. The removal of S_1 from G makes the graph disconnected and the induced graph of $G - S_1$ has exactly two components. Since by Observation 1.5, $\kappa(G) \leq \kappa_3(G)$ we need to remove few vertices from $G - S_1$ to make the graph disconnected into at least three components. Since $d(p_1 \times p_2^2) = d(p_1^2 \times p_2) = 1$ and its neighbor say $N(p_1 \times p_2^2) = p_1^2$ also $N(p_1^2 \times p_2) = p_2^2$. Hence either the set $S = \{S_1 \cup p_1^2\}$ or $S = \{S_1 \cup p_2^2\}$ is a three component cut. Since no subset of S is a 3-component cut, S is minimum. Hence we have $\kappa_3(G) = |S| = 3$ □

Theorem 2.5. *Let $G = V_n$ be an arithmetic graph, $n = p_1^{a_1} \times p_2^{a_2}$ where $a_1 > 2, a_2 \geq 2$ then $\kappa_3(G) = 2$.*

Proof. By Theorem1.2, G is a bipartite graph with partitions A and B. The partition A consists of prime vertices, power of prime vertices. Also, the partition B consists of product of primes vertices, product of power of prime vertices. If $a_1 > 2$ and $a_2 \geq 2$ then the number of vertices in B, which are adjacent only to p_1 and p_2 is at least two. So the removal of the vertices in $S = \{p_1, p_2\}$ from G makes the induced graph $G[V - S]$ disconnected into at least three components. Hence the set S is a 3-component cut. Also, by Theorem 1.6 $\kappa(G) = 2$ shows that the set S is minimum. Hence $\kappa_3(G) = |S| = 2$. □

Theorem 2.6. *For an arithmetic graph $G = V_n, n = p_1 \times p_2 \times p_3$ the 3-component connectivity number $\kappa_3(G) = 4$.*

Proof. By Theorem 1.3, $|V| = 7$ and the vertex set $V(G) = \{p_1, p_2, p_3, p_1 \times p_2, p_1 \times p_3, p_2 \times p_3, p_1 \times p_2 \times p_3\}$. In this graph if we remove the adjacent vertices $S_1 = \{p_1, p_2, p_3\}$ of a minimum degree vertex $p_1 \times p_2 \times p_3$ the induced graph $G[V - S_1]$ has two components G_1 and G_2 where G_1 is an isolated vertex $p_1 \times p_2 \times p_3$ and G_2 is a complete graph k_3. The removal of any vertex from $G[V - S_1]$ does not make the graph disconnected into three components. On the other hand, if we remove the set $S = \{p_1 \times p_2, p_1 \times p_3, p_2 \times p_3, p_1 \times p_2 \times p_3\}$ the graph gets disconnected into three components, each component is an isolated vertex. Since no proper subset of S satifies the definition of 3-component cut S is minimum. Hence $\kappa_3(G) = 4$. □

Theorem 2.7. *For an arithmetic graph $G = V_n, n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, r > 3$ and $a_i = 1, \forall\ i \in \{1, 2, \ldots, r\}$. Then $\kappa_3(G) = 2r - 1$.*

Proof. Consider an arithmetic graph $G = V_n,\ n = p_1 \times p_2 \times \cdots \times p_r$ and $r > 3$. Then the vertex set consists of primes, product of two primes, product of three primes,....., product of $r-1$ primes, product of r primes. By Theorem 1.7, we know that $\kappa(G) = r$, if we remove the set $S_1 = \{p_1, p_2, \ldots, p_r\}$ from G then the graph $G[V(G) - S_1]$ has two components G_1 and G_2 where G_1 is an isolated vertex

$p_1 \times p_2 \times \cdots \times p_r$ and G_2 is a connected graph. Hence $\kappa_3(G) > \kappa(G)$. Let $X_1 = \{\prod_{i \in B^*} p_i : B^* \subset \{1, 2, \ldots, r, \ |B^*| = r-1\}$ be the set of minimum degree vertices in the component G_2. Choose any one of the vertex $u \in X_1$ let it be $u = p_1 \times p_2 \times \cdots \times p_{r-1}$, the degree $d(u) = r-1$ and $N(u) = \{p_1 \times p_r, p_2 \times p_r, \ldots, p_{r-1} \times p_r\}$. The set $S = \{p_1, p_2, \ldots, p_r, p_1 \times p_r, p_2 \times p_r, \ldots, p_{r-1} \times p_r\}$ is a 3-component cut. Since $G[V - S]$ has exactly three components $G_1,\ G_2,\ G_3$ where G_1 is an isolated vertex $p_1 \times p_2 \times \cdots \times p_r$, G_2 is an isolated vertex $p_1 \times p_2 \times \cdots \times p_{r-1}$ and G_3 is a connected component containing vertices such as product of two primes other than the vertices in S, product of three primes,.....,product of $r-2$ primes, $X_1 - \{p_1 \times p_r, p_2 \times p_r \ldots p_{r-1} \times p_r\}$. To prove S is minimum, suppose $S' \subset S$ be a minimum 3-component cut then $G[V - S']$ has at least three components $G_1,\ G_2,\ G_3$ where G_1 is an isolated vertex $p_1 \times p_2 \times \cdots \times p_r$, G_2 is an isolated vertex $p_1 \times p_2 \times \cdots \times p_{r-1}$ and G_3 is a connected component. Let us assume that $v \in S$ and $v \notin S'$ if $v = p_i,\ i \in \{1, 2, \ldots, r\}$. Since G_1 is an isolated vertex $p_1 \times p_2 \times \cdots \times p_r$ then the $(p_1 \times p_2 \times \cdots \times p_r, p_i) \neq p_i$ which is a contradiction to the definition of an arithmetic graph. Similarly if $v = p_i \times p_{r-1}$ then we have $(p_1 \times p_2 \times \cdots \times p_{r-1}, p_i \times p_{r-1}) \neq p_i$ which is a contradiction. Therefore S is minimum and by Theorem 1.4, we have $\kappa_3(G) = |S| = d(p_1 \times p_2 \times \cdots \times p_r) + d(p_1 \times p_2 \times \cdots \times p_{r-1}) - |W|$.
$= r + 2(r-1) - (r-1) = 2r - 1$, where $|w|$ is the number of vertices which are adjacent to both $p_1 \times p_2 \times \cdots \times p_r$ and $p_1 \times p_2 \times \cdots \times p_{r-1}$. □

The following example shows that for any arithmetic graph $G = V_n,\ n = p_1^{a_1} \times p_2^{a_2}$ where $a_1 > 2, a_2 \geq 2$ the 3-component connectivity number $\kappa_3(G)$ and the connectivity number $\kappa(G)$ are equal.

Example 2.1. *The following Figure 1 shows an arithmetic graph $G = V_{72},\ 72 = 2^3 \times 3^2$. Clearly the vertex set consists of vertices $V(G) = \{2, 2^2, 2^3, 3, 3^2, 2 \times 3,\ 2^2 \times 3, 2^3 \times 3,\ 2 \times 3^2, 2^2 \times 3^2,\ 2^3 \times 3^2\}$. The minimum degree $\delta(G) = 2$ and the 3-component cut $S = \{2, 3\}$.*

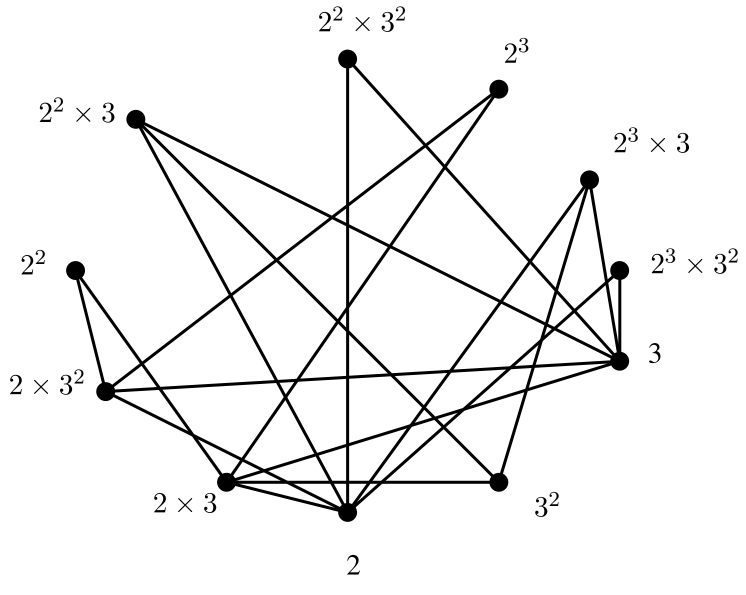

Figure 2.1: Arithmetic graph $G = V_{72}$

The induced graph $G[V_{72} - S]$ shown in Figure 2, has three components $G_1,\ G_2$ and G_3 where G_1 is a connected component, G_2 and G_3 are isolated vertices $2^3 \times 3^2$ and $2^2 \times 3^2$ respectively. By Theorem 1.6, the vertex cut is also $\{2, 3\}$. This shows that $\kappa_3(G) = \kappa(G)$ as well as the vertices in 3-component cut is same as the vertices in the vertex cut.

Theorem 2.8. *For an arithmetic graph $G = V_n, n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, r \geq 3, a_1 = 2$ and $a_j = 1, \forall\ j \in \{2, \ldots, r\}$. Then $\kappa_3(G) = r + 1$.*

Proof. Consider an arithmetic graph $G = V_n$ where $n = p_1^2 \times p_2 \times \cdots \times p_r,\ r \geq 3$. Then the graph G contains vertices such as $p_1, p_2, \ldots, p_r, p_1^2, p_1 \times p_2, \ldots, p_1 \times p_r, p_1^2 \times p_2, \ldots, p_1^2 \times p_2, p_2 \times p_3, \ldots p_{r-1} \times$

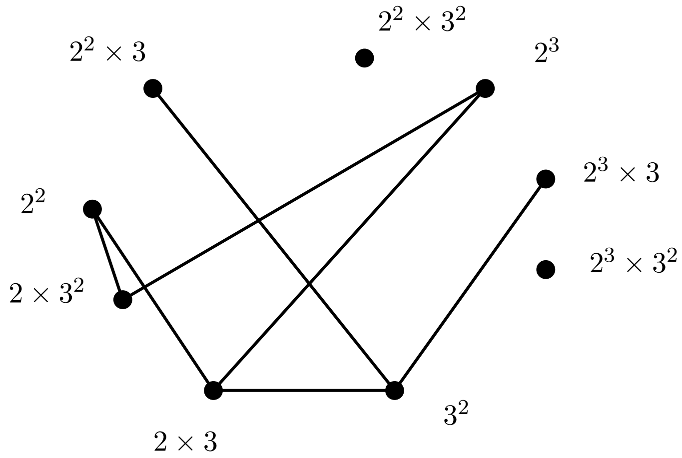

Figure 2.2: Induced graph $G[V_{72} - S]$

$p_r, \ldots, p_1 \times p_2 \times \cdots \times p_r, p_1^2 \times p_2 \times \cdots \times p_r$. By Theorem 1.8 the connectivity number $\kappa(G) = r$ and the minimum vertex cut is $S_1 = \{p_1, p_2, \ldots, p_r\}$. The induced graph $G[V - S_1]$ has exactly two components G_1 and G_2 where G_1 is an isolated vertex $p_1^2 \times p_2 \times \cdots \times p_r$ and G_2 is a connected graph which contains all the vertices other than $V(G) - \{r+1\}$ vertices. Now the minimum degree vertex in the connected component G_2 is $p_1 \times p_2 \times \cdots \times p_r$ and $N(p_1 \times p_2 \times \cdots \times p_r) = p_1^2$. So, the removal of the set $S = S_1 \cup \{p_1^2\}$ from G makes the graph disconnected into exactly three components. Hence the set $S = \{p_1, p_2, \ldots, p_r, p_1^2\}$ is a 3- component cut. Since the removal of any vertex from the set S, either violates the connectivity property or the number of components in the induced graph is less than three. Therefore the set S is minimum. Hence we have $\kappa_3(G) = |S| = r + 1$. □

Theorem 2.9. *If $G = V_n$ is an arithmetic graph $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, r \geq 3, a_i > 2$ for at least one i then $\kappa_3(G) = r$.*

Proof. Let $G = V_n$ be an arithmetic graph where $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, r \geq 3$, **Case(i)** Let us assume that $a_1 > 2$ and $a_j = 1$ for $j \in \{2, 3, \ldots, r\}$. The vertex set $V(G) = \{p_1^{\alpha_1}, p_2, \ldots, p_r, p_1^{\alpha_1} \times p_j, p_{r-1} \times p_r \ldots, p_1^{\alpha_1} \times p_2 \times \cdots \times p_r; 1 \leq \alpha_1 \leq a_1, j = 1, 2 \ldots r\}$. Here the vertices $\{p_1^{\alpha_1} \times p_2 \times \cdots \times p_r; 1 \leq \alpha_1 \leq a_1\}$ has degree r and have unique neighbors say $S = \{p_1, p_2, \ldots, p_r\}$. So, the removal of S from G results the graph disconnected and since $a_i > 2$ the number of isolated vertices in $G[V - S]$ is at least two and a connected component. Thus the set S satisfies the 3-Component cut definition, hence S is a 3-component cut. Also, By Theorem1.8 $\kappa(G) = r$, this shows that S is a minimum 3- component cut. Hence proved.

Case(ii) If $a_i > 2$ for more than one i. Let us assume that $n = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}$ such that $a_1 \geq a_2 \geq \ldots a_r$. By the definition of an arithmetic graph, we know that $(p_1^{\alpha_1} \times p_2^{a_2} \times \cdots \times p_r^{a_r}, p_i) = p_i$; for $i \in \{1, 2, \ldots, r\}; 2 < \alpha_1 \leq a_1$. Since, $a_1 > 2$ the number of vertices in G which are adjacent only to $S = \{p_1, p_2, \ldots, p_r\}$ is at least two. Thus the induced graph G[V-S] has at least three components. Hence by the proof of Theorem 2.9 follows the required result. □

3 Conclusion

From the above study, we observe that for an arithmetic graph $G = V_n$, if the number of primes in n is greater than two then the 3-component connectivity number is strictly greater than its connectivity number. But if the number of primes in n is greater than two and at least one of its prime power is greater than two then the connectivity number and 3-component connectivity number are same which is equal to number of primes in n.

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